

**THREE-DIMENSIONAL INELASTIC ANALYSIS METHODS FOR HOT SECTION COMPONENTS**

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Under a two-year program at the General Electric Company, a series of three-dimensional inelastic structural analysis computer codes were developed and delivered to NASA.<sup>1</sup> A summary of the present state of the capabilities of these computer codes is presented here.

The objective of this program was to develop analytical methods capable of evaluating the cyclic time-dependent inelasticity which occurs in hot section engine components. Because of the large excursions in temperature associated with hot section engine components, the techniques developed must be able to accommodate large variations in material behavior including plasticity and creep (ref. 1). To meet this objective, General Electric developed a matrix consisting of three constitutive models and three element formulations. A separate program for each combination of constitutive model - element model was written, making a total of nine programs. The source codes of the nine programs range in size from 7300 lines for the Bodner/twenty node to 19 000 lines for the Haisler and Allen/nine node. Table I shows the length of each source code. All of the codes were given a stand-alone capability of performing cyclic nonlinear analysis.

The three constitutive models consist of a simple model, a classical model, and a unified model. In an inelastic analysis, the simple model uses a bilinear stress-strain curve to determine the plastic strain and a power law equation to obtain the creep strain. The second model is the classical model of Haisler and Allen (ref. 2). The third model is the unified model of Bodner, Partom, and Partom (ref. 3). The attributes of the three constitutive models are listed in Table II. All of the models were programmed for a linear variation of loads and temperatures with the material properties being temperature dependent.

The three element formulations used are an eight-node isoparametric shell element, a nine-node shell element, and a twenty-node isoparametric solid element. The eight-node element uses serendipity shape functions for interpolation and Gaussian quadrature for numerical integration. Lagrange shape functions are used in the nine-node element. For numerical integration, the nine-node element uses

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Simpson's rule. The twenty-node solid element uses Gaussian quadrature for integration. The other attributes of these elements are listed in Table III.

For the linear analysis of structures, the nine codes use a blocked-column skyline, out-of-core equation solver. To analyze structures with nonlinear material behavior, the codes use an initial stress iterative scheme. Aitken's acceleration scheme was incorporated into the codes to increase the convergence rate of the iteration scheme.

The ability to model piecewise linear load histories was written into the codes. Since the inelastic strain rate can change dramatically during a linear load history, a dynamic time-incrementing procedure was included. The maximum inelastic strain increment, maximum stress increment, and the maximum rate of change of the inelastic strain rate are the criteria that control the size of the time step. The minimum time step calculated from the three criteria is the value that is used.

In dynamic analysis, the eigenvectors and eigenvalues can be extracted using either the determinant search technique or the subspace iteration method. These methods are only included with those finite-element codes containing the eight-node shell element.

The nine codes have been compiled on the CRAY-1 machine at NASA Lewis Research Center. Table IV shows those features contained in the codes that have been checked. Additional work has to be done in examining the other features contained in these codes. Also, test problems need to be analyzed so that the performance of these codes can be examined.

#### REFERENCES

1. McKnight, R.L.; Chen, P.C.; Dame, L.T.; Holt, R.V.; Huang, Ho; Hortle, M.; Gellin, S.; Allen, D.H.; and Haisler, W.E.: 3-D Inelastic Analysis Methods for Hot Section Components, Turbine Engine Hot Section Technology - 1986, NASA CP-2444, pp. 257-268.
2. Allen, D.H.; and Haisler, W.E.: A Theory of Thermoplastic Materials, Computers and Structures, Vol. 13, pp. 129-135, 1981.
3. Bodner, S.A.; Partom, I.; and Partom, Y.: Uniaxial Cyclic Loading of Elastic-Viscoplastic Material, ASME J. Appl. Mech., Vol. 46, p. 805, 1979.

TABLE I. - APPROXIMATE LENGTH OF THE NINE  
SOURCE CODES

Constitutive models	Element formations		
	20-Node	8-Node	9-Node
	Length of code, number of lines		
Simple	8300	13 800	17 900
Classical	9200	16 300	19 000
Unified	7300	13 800	17 600

TABLE II. - CONSTITUTIVE MODELS

Simple model	Classical model	Unified model
Uncoupled plasticity and creep	Uncoupled plasticity and creep	-----
Plasticity	Plasticity	No yield surface
Isotropic hardening	Combined isotropic and kinematic hardening	Isotropic hardening
Piecewise linear stress-strain curves	Piecewise linear stress-strain curves	-----
Prandtl-Reuss flow rule	Modified Prandtl-Reuss flow rule	-----
Nonisothermal	Nonisothermal	Nonisothermal
Creep Steady state Isotropic hardening Prandtl-Reuss flow rule Nonisothermal	Creep Steady state Isotropic hardening Prandtl-Reuss flow rule Nonisothermal	Second-order Adams- Moulton rule used to carry out integration

TABLE III. - ELEMENT FORMULATION

8-Node shell	9-Node shell	20-Node solid
Five degrees of freedom 3 Displacements 2 Rotations	Five degrees of freedom 3 Displacements 2 Rotations	Three degrees of freedom 3 Displacements
Serendipity shape functions	LaGrange shape functions	-----
No rotational stiffness about the normal to the mid-surface; deleted prior to assembly	Rotation about the normal to the mid-surface is treated as a prescribed displacement	-----
Isotropic or orthotropic elastic properties	Isotropic or orthotropic elastic properties	Isotropic or orthotropic elastic properties
Surface, line, nodal, RPM, thermal, and gravity loads	Surface, line, nodal, RPM, thermal, and gravity loads	Surface, nodal, RPM, thermal, and acceleration loads
Prescribed displacements	Prescribed displacements	Prescribed displacements
Gaussian quadrature used for numerical integration	Simpson's rule used for numerical integration	Gaussian quadrature used for numerical integration

TABLE IV. - CODE FEATURES<sup>a</sup>

Feature	Simple model			Classical model			Unified model		
	8-Node	9-Node	20-Node	8-Node	9-Node	20-Node	8-Node	9-Node	20-Node
Free format data input	X <sup>a</sup>	X	X	X	X	X	X	X	X
Global coordinate system			---						
Cartesian	X	X	X	X	X	X	X	X	X
Spherical <sup>b</sup>	---	---	---	---	---	---	---	---	---
Cylindrical <sup>b</sup>	---	---	---	---	---	---	---	---	---
Local coordinate system									
Cartesian	X	X	X	X	X	X	X	X	---
Spherical <sup>b</sup>	---	---	---	---	---	---	---	---	---
Cylindrical <sup>b</sup>	---	---	---	---	---	---	---	---	---
Automatic generation of nodal coordinates	---	---	N/A <sup>c</sup>	---	---	N/A	---	---	N/A
Automatic generation of element connectivities	X	X	N/A	X	X	N/A	X	X	N/A
Restart capability	---	---	---	---	---	---	---	---	---
Dynamic allocation	X	X	X	X	X	X	X	X	X
Blocked column skyline equation solver	X	X	X	X	X	X	X	X	X
Initial stress iterative scheme	---	---	---	---	---	---	---	---	---
Aitken's acceleration scheme	---	---	X	---	---	X	---	---	X
Dynamic time incrementing	---	---	X	---	---	X	---	---	X
Convergence criteria									
Effective plastic strain	---	---	X	---	---	X	---	---	X
Effective stress	---	---	X	---	---	X	---	---	X
Dynamic analysis									
Eigenvalue	X	N/A	N/A	X	N/A	N/A	X	N/A	N/A
Eigenvector	X	N/A	N/A	X	N/A	N/A	X	N/A	N/A
Material change option	---	---	---	---	---	---	---	---	---
Element removal option	---	---	---	---	---	---	---	---	---
Loads	X	X	X	X	X	X	X	X	X
Midsized node generation	---	---	---	---	---	---	---	---	---
Skewed coordinate system	X	X	X	X	X	X	X	X	X
Orthotropic orientation definition	X	X	X	X	X	X	X	X	X

<sup>a</sup>X = Feature has been checked.<sup>b</sup>Cylindrical and spherical coordinate systems have not been checked.<sup>c</sup>N/A = Not a feature of this particular code.